Evaluation of Optical Model Potential Using Neutron Induced Cross Section Reaction for Spherical Uranium-238 Isotope up to 20MeV

Iman Tarik AL-Alawy* Ronak Ikram Ali

Al-Mustansiriya University, College of Science, Physics department, Baghdad - Iraq

*Email: drimantarik@yahoo.com

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ABSTRACT. The evaluation are based on mainly on the calculations of the nuclear optical model potential and relevant parameters are collected and selected from References Input Parameter Library (RIPL) which is being developed under the international project coordinated by the International Atomic Energy Agency (IAEA). The analyzing of a complete energy range has done starting from threshold energy for each reaction. The cross sections are reproduced in fine steps of incident neutron energy with 0.01MeV intervals with their corresponding errors. The recommended cross sections for available experimental data taken from EXFOR library have been calculated for all the considered neutron induced reactions for U-238 isotopes. The calculated results are analyzed and compared with the experimental data. The optimized optical potential model parameters give a very good agreement with the experimental data over the energy range 0.001-20MeV for neutron induced cross section reactions \((n,f)\), \((n,tot)\), \((n,el)\), \((n,inl)\), \((n,2n)\), \((n,3n)\), and \((n,\gamma)\) for spherical U-238 target elements.

1. INTRODUCTION

The excitation functions in induced neutron nuclear reactions \((n,f)\), \((n,tot)\), \((n,el)\), \((n,inl)\), \((n,2n)\), \((n,3n)\), and \((n,\gamma)\) measured for U-238 with the aid of EXFOR library. The cross sections for these reactions have been evaluated in the present work for the exact estimation of the cross sections among different authors. This paper describes the standard optical model potential analyses of the spherical U-238 target elements up to 20MeV. The present paper also describes the background of the References Input Parameter Library (RIPL) used for input parameters. These data are used in the real and imaginary part of optical model potential-special emphasis is placed in this study on the isotope dependence of the optical model potential.

2. RECOMMENDED CROSS SECTION

The available measured data from EXFOR library for the cross section \((n,f)\), \((n,tot)\), \((n,el)\), \((n,inl)\), \((n,2n)\), \((n,3n)\) and \((n,\gamma)\) reactions for U-238 respectively have been plotted interpolated and recalculated in different fine steps and for different energy range of incident neutron by using Matlab-8.0 in order to calculate the recommended cross section for each mentioned reactions within the following steps with a minimum \(\chi^2\) for U-238 neutron induced reaction is equal to 0.0006.

1- The interpolation for the nearest data for each energy interval as a function of cross sections and their corresponding errors have been done using Matlab-8.0.

2- The sets of experimental cross sections data are collected for different authors and with different energy intervals. The cross sections with their corresponding errors for each value are re-arranged according to the energy interval 0.01 MeV for available different energy range for each author.

3- The normalization for the statistical distribution of cross sections errors to the corresponding cross section values for each author has been done.

4- The interpolated values are calculated to obtain the adopted cross section which is based on the weighted average calculation according to the following expressions [1].
\[ \sigma_{w,a.} = \frac{\sum_{i=1}^{n} (\Delta \sigma_i \sigma_i)^2}{\sum_{i=1}^{n} (\Delta \sigma_i)^2} \]

(1)

Where the standard deviation error is:

\[ S.D. = \frac{1}{\sqrt{\sum_{i=1}^{n} (\Delta \sigma_i)^2}} \]

(2)

Where \( \sigma_i \): is the cross section value.

\( \Delta \sigma_i \): is the corresponding error for each cross section value.

Figs. 1 to 4 illustrate the recommended cross sections for the above mentioned reactions as calculated in the present work compared with EXFOR library. It can be seen that than in the footnote of each figure, the refry of authors name are arrange according to the year of measured data are listed with the present calculated recommended cross section. The results are in good agreement with the measured data.

3. OPTICAL MODEL POTENTIAL

In the frame of the optical model, all the interactions between the nucleons of the projectile and the nucleons of the target are replaced by an average and central interaction \( V(r) \) between the projectile and the target in their ground states. The nuclear optical model used to describe the interaction between two nuclei is inspired by the optical phenomenon. The nuclear medium diffracts one part of the incident wave which models the incident particle and another part of the wave is refracted [2]. As the nucleon-nucleon interaction is a short range interaction, the potential \( V_r \times f_r (r, r, a_r) \), which is approximately the sum of nucleon-nucleon interactions, has the same behavior. The nucleons in the core of the nucleus undergo only the interaction with their closest neighbors. Due to this saturation of the nuclear forces, \( V_r \times f_r (r, r, a_r) \) is uniform inside the nucleus and then decreases exponentially in the surface region [3].

4. BACKGROUND OF (RIPL)

The Reference Input Parameter Library (RIPL) is being developed under the international project coordinated by the International Atomic Energy Agency (IAEA). The practical use of nuclear reactions requires a considerable numerical input that describes properties of the nuclei and interactions involved. The (RIPL) represents a fairly comprehensive set of such parameters, collected and selected from sources all over the world. The (RIPL) contains input parameters for theoretical calculations of nuclear reaction cross sections. The library is targeted at users of nuclear reaction interested in nuclear applications. The main recommended optical model parameters files in the (RIPL) are Los Alamos (U.S.A.), Beijing (China), and Jaeri (Japan) files [4].

5. THEORETICAL BASIS OF OPTICAL MODEL POTENTIAL

The present evaluations are based mainly on the calculations the optical model potential. A standard form of the optical model potential and relevant parameters used in the present work contains volume, surface, and spin-orbit parts, each having real and imaginary components. This potential can be written as follows [4,5,6]:

\[ V(r,E) = -V_r \times f_r (r, r, a_r) \]

\[ + i \left\{ 4 \times a_d \times W_d \times \left[ \frac{df_d(r, r, a_d)}{dr} \right] - W_g \times \exp(-X_g^2) - W_s \times f_s (r, r, a_r) \right\} \]

\[ + \frac{\hbar^2}{r} \times \left[ \frac{df_{so}(r, r_{so}, a_{so})}{dr} \right] + i W_{so} \times \left[ \frac{df_{so}(r, r_{so}, a_{so})}{dr} \right] \times \left( \ell + s \right) \]

(3)
In equation (3) $v_r$ and $w_r$ are the real and imaginary volume potential well depths, $w_g$ is the well depth for the surface derivative term, $w_d$ is the well depth for the global nucleon-nucleon optical potential, $v_{so}$ and $w_{so}$ are the real and imaginary well depths for the spin-orbit potential, and $\lambda_i^2$ is the pion Compton wavelength squared ($\approx 2$). The quantity $C_{ij}$ is the well depth for the global nucleon potential as a function of radial distance and the energy of induced neutron for spherical U-238 target elements.

$$\vec{\ell} \cdot \vec{s} = \ell \quad \text{for } j = \ell + \frac{1}{2}$$  \hspace{1cm} (4)

$$\vec{\ell} \cdot \vec{s} = -(\ell + 1) \quad \text{for } j = \ell - \frac{1}{2}$$ \hspace{1cm} (5)

The $f_i(r, r_i, a_i)$ are radial-dependent form factors. The real potential, imaginary potential and form factors are defined below [4]:

1. **Real Potential**

$$v_r, v_{so}$$ are the depths of real potential in (MeV).

$$v_i = v_{r0} + v_{r1} \times E + v_{r2} \times E^2 + (v_{r3} + v_{r4} \times E) \times (N - Z) / A \quad \text{with } i = r, so$$  \hspace{1cm} (6)

Where $v_{r0}$, $v_{r1}$, $v_{r2}$, $v_{r3}$, $v_{r4}$ and $v_{so0}$, $v_{so1}$, $v_{so2}$, $v_{so3}$, $v_{so4}$ are the depth parameters of real potential in (MeV) taken from (RIPL). $Z$, $N$, and $A$ are the numbers of protons, neutrons and nucleons in the target nuclide respectively. $E$ is the energy of incident particle. (Hint: We select the energy at maximum cross section for different authors for selected reactions).

2. **Imaginary Potential**

$$w_d, w_v, w_g, w_{so}$$ are the depths of imaginary potential in (MeV).

$$w_i = w_{d0} + w_{d1} \times E + w_{d2} \times E^2 + (w_{d3} + w_{d4} \times E) \times (N - Z) / A \quad i = d, v, g, so$$  \hspace{1cm} (7)

Where $w_{d0}$, $w_{d1}$, $w_{d2}$, $w_{d3}$, $w_{d4}$, $w_{v0}$, $w_{v1}$, $w_{v2}$, $w_{v3}$, $w_{v4}$, $w_{g0}$, $w_{g1}$, $w_{g2}$, $w_{g3}$, $w_{g4}$, and $w_{so0}$, $w_{so1}$, $w_{so2}$, $w_{so3}$, $w_{so4}$ are the depth parameters of imaginary potential in (MeV) taken from (RIPL).

3. **Form Factor**

Wood–Saxon form factors is permitted for $f_i(r, r_i, a_i)$ terms in equation (3), as follows:

$$f_i(r, r_i, a_i) = \frac{1}{1 + \exp(X_i)} \quad \text{with } i = r, d, v, so \quad \text{(Wood–Saxon form factor)}$$  \hspace{1cm} (8)

Where

$$X_i = (r - R_i) / a_i \quad \text{with } i = r, d, g, v, so$$ \hspace{1cm} (9)

$r$ is the radial distance in (fm). The nuclear radius $R_i$ is given by:

$$R_i = (r_{i0} + r_{i1} \times E) \times A^{\frac{1}{3}} + C_i$$ \hspace{1cm} (10)

And the form used for the diffuseness, $a_i$, is given by:

$$a_i = a_{i0} + a_{i1} \times E$$ \hspace{1cm} (11)

Where:

$$r_0$$, $$r_{r0}$$, $$r_{r1}$$, $$C_r$$, $$a_{r0}$$, $$a_{r1}$$; $$r_d$$, $$r_{d0}$$, $$r_{d1}$$, $$C_d$$, $$a_{d0}$$, $$a_{d1}$$; $$r_g$$, $$r_{g0}$$, $$r_{g1}$$, $$C_g$$, $$a_{g0}$$, $$a_{g1}$$

$$r_v$$, $$r_{v0}$$, $$r_{v1}$$, $$C_v$$, $$a_{v0}$$, $$a_{v1}$$; $$r_{so}$$, $$r_{so0}$$, $$r_{so1}$$, $$C_{so}$$, $$a_{so0}$$, $$a_{so1}$$

are the geometry parameters of real potential in (fm) taken from (RIPL). The optical potential program has been built in the present work using Matlab-8.0. The aim of this program is to calculate the real and imaginary optical potential as a function of radial distance and the energy of induced neutron for spherical U-238 target elements.
6. CALCULATED RESULTS AND DISCUSSION

A limited number of parameters for spherical potential are included for incident neutron particles. The energy dependence of the neutron potential based on the Uranium isotopes (Z=92, A=238) is E=0.001-20MeV for spherical U-238 nuclei. Which are included in the present calculations to cover the same energy range for the same target charge and mass. The optical model potential (OMP) parameters are including in the (RIPL) optical file coordinated research project with Beijing Library. The parameters for optical model potential used in this work are tabulated in table 1 for spherical U-238 target element. The global potentials are calculated for systematics utilization of nuclear radial distance r=1 to 20fm as well as real and imaginary potential. This model represents the scattering in terms of a complex potential V(r,E), see Eq. 3, where the functions V and W are selected to give the potential its proper radial dependence. The real part, V, is responsible for the elastic scattering it describes the ordinary nuclear interaction between target and projectile and may therefore be very similar to a shell model potential. The imaginary part, W, is responsible for the absorption. The usual optical model parameters for Uranium-238 (spherical nucleus) has real optical depth $V_{r_0}$ is of the order of 49.7237MeV with radius of the real depth $r_{r0}=1.2684$fm with spin potential $V_{so}=6.2000$MeV with radius $r_{so}=1.2684$fm, and imaginary optical depth $W_{do}$ is 6.5060MeV and $W_{so}=1.4504$MeV with radius $r_{so}=1.3059$fm and $r_{o}=1.3059$fm with energy range(0.001-20MeV). The radial distance is to be at most of the order of the r=1:20fm and the energy of incident neutrons have been taken at maximum cross section. All parameters used in this work for optical model potential have been taken from (Beijing) library [32]. Table 1 shows these parameters for Uranium-235. Fig. 8 shows the optical model potential for Uranium target element (U; A = 238) induced by neutron, in which absorption W is relatively weaker than elastic scattering V. The absorptive part, W, at low energies must have a very different form. It is clear from these figures that for spherical structure both absorption and scattering depth parameters for Uranium-238 are greater than that for Uranium-238, since the well depth depend on the mass number of the target element. Because of the exclusion principle, the tightly bound nucleons in the nuclear interior cannot participate in absorb the relatively low energy carried by the incident particle. The optical potential is thus often has the proper shape of being large only near the surface, as shown in fig. 8. At higher energy, where the inner nucleons can also participate in absorption, W, may look more like V. A spin – orbit term is also included in this optical potential. It is also peaked near the surface, because the spin density of the inner nucleons vanishes. A Wood – Saxon form factor is also included. The calculation using the optical model potential, as described in this work, does not deal with where the absorbed particles actually go; they simply disappear from the elastic channel.

7. CONCLUSIONS

We have evaluated the neutron induced nuclear cross section data of spherical Uranium-238 isotope for considerable energy range. The recommended cross sections are in good agreement with experimental data. The reliability in this work is to estimate the global optical parameters chosen for the energy range 0.001-20MeV from Beijing library for spherical U- 238 target elements of neutron induced reactions. The results confirm that the global optical potential parameters are appropriate for these calculations. Hence, the optical model potential is successful in accounting for neutron induced reactions and leads to an understanding of the nucleon-nucleon interactions.
Fig. 1. Left side: The recommended cross section of the $^{238}_{92}U(n,f)$ reaction as calculated by the present work compared with EXFOR library. Right side: The recommended cross section of the $^{238}_{92}U(n,tot)$ reaction as calculated by the present work compared with EXFOR library. Data in left side: Data1: Manabe et al. (1988) [7]. Data2: Meadows et al. (1988) [8], Data3: Shcherbakov et al. (2001) [9], Data4: Khattab (2007) [10], Data5: Nolte et al. (2007) [11]. Data6: Present work (PW). Data in right side: Data1: Hayes et al. (1973) [12], Data2: Poenitz et al. (1981) [13], Data3: Harvey et al. (1988) [14], Data4: Abfalterer et al. (2001) [15], Data5: Present work (PW).

Fig. 2. Left side: The recommended cross section of the $^{238}_{92}U(n,e1)$ reaction as calculated by the present work compared with EXFOR library. Right side: The recommended cross section of the $^{238}_{92}U(n,inl)$ reaction as calculated by the present work compared with EXFOR library. Data in left side: Data1: Batchelor et al. (1965) [16], Data2: Barnard et al. (1966) [17], Data3: Haouat et al. (1982) [18], Data4: Present work (PW). Data in right side: Data1: Glazkov (1963) [19], Data2: Kegel et al. (1997) [20], Data3: Present work (PW).

Fig. 3. Left side: The recommended cross section of the $^{238}_{92}U(n,2n)$ reaction as calculated by the present work compared with EXFOR library. Right side: The recommended cross section of the $^{238}_{92}U(n,3n)$ reaction as calculated by the present work compared with EXFOR library. Data in left side: Data1: Landrum et al. (1973) [21], Data2: Chou (1978) [22], Data3: Veeser and Arthur (1978) [23], Data4: Frehaut et al. (1980) [24], Data5: Raics et al. (1990) [25], Data6: Konno et al. (1993) [26], Data7: Filatenkov et al. (1999) [27], Data8: Present work (PW). Data in right side: Data1: Veeser and Arthur (1978) [23], Data2: Zhou (1978) [28], Data3: Frehaut et al. (1980) [24], Data4: Present work (PW).
Fig. 4. The recommended cross section of the $^{238}_{92}\text{U}(n,\gamma)^{239}_{92}\text{U}$ reaction as calculated by the present work compared with EXFOR library. Data1: Ryves et al. (1973) [29]. Data2: Poenitz et al. (1981) [13]. Data3: Macklin et al. (1988) [30]. Data4: Voignier et al. (1992) [31]. Data5: Present work (PW).

Table 1 Parameters for optical model potential used for spherical Uranium-238 in Beijing Library with energy range (0.001-20) [32].

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Fig. 5. The Optical Model Potential of neutron induced reaction on spherical U-238 calculated in the present work as a function of radial distance. Typical parameters chosen are taken from energy range (0.001 - 20 MeV) (Beijing Library).
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