The effect of cosmic vacuum on the properties of scalar field

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ABSTRACT. The thesis explains the effect of cosmic vacuum of gravitational field on properties of scalar field equation. In the space-time plane, the scalar field equation has periodic solution $\varphi(x,y,z,t) = A\cos(\pm k x + \omega t)$. Considering the cosmic vacuum of gravitational field (using De Sitter metric in its synchronization time) the equation of scalar field will have accurate solution in form of Bessel function.

By using the asymptotic representation, the periodic solution ($t \to \pm \infty$) will vanish. The scalar field equation when $t \to +\infty$ will decrease regularly, and when $t \to -\infty$ it will increasing fluctuate.

1. INTRODUCTION

The study of cosmic vacuum is one from import lines studying in modern cosmology. Cosmological observations shows presence cosmic vacuum in universe [2]. Vacuum create anti-gravitation field, which call for acceleration cosmological expansion. The detection of cosmic vacuum made substantial evolution on the modern’s nature of universe.

2. THE DISCOVERY OF COSMIC VACUUM

In cosmological science occur developments that many specialists consider it revolution, thanks only three points in cosmology:

a. In the universe dominates vacuum of energy density exceeds all the "usual" forms of cosmic matter together.

b. Antigravity governs the dynamics of the cosmological expansion.

c. Cosmological expansion is accelerating, and the space in four-dimensional space-time is static, because the time a static [2].

Discovery made by astronomer’s observers, who have been studying distant supernovae.

Observers have data on only a few supernovae, but already it was enough to notice the cosmological effect of decrease in the apparent brightness with the distance [9]. More precisely, it is better to look not at a distance, but a redshift “as is usually done in the case of distant sources”. It was found that the decrease in the average brightness is faster than that expected for the cosmological theory, which has recently been considered the standard [4].

The theory of the expanding universe was created by Friedman in 1922-1924 [1]. With the opening of the cosmological expansion all doubts in the introduction of the cosmological constant disappeared.

The geometry of Friedman’s four-dimensional space describes the metric element

$$ds^2 = c^2 dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \quad (1)$$
Friedman’s theory assumes that the distribution of matter is homogeneous in the universe. In his theory Friedman predicts the cosmological expansion in homogeneous and isotropic space must occur according linear law [6]: “in any moment the velocity distant source, which exist at distance \( R \) proportion to this distance”.

\[
V = HR, 
\]

\( H \)- constant coefficient, \( H = \sqrt{\frac{A}{3}} \approx 10^{-28} \frac{1}{cm} \).

Vacuum appear in cosmology with Einstein’s cosmological constant. In the first role cosmological constant became how to explain anti-gravity. Einstein predicts in that way possible equilibrium gravity material’s universe, and i.e. the universe itself stationary [12].

3. THE SCALAR FIELD

Scalar field explains particles with spin \( s=0 \). Effective (pseudo)scalar field explains natural spinless mesons. Complex (pseudo)scalar field explains charge spinless mesons [11]. The difference between scalar and pseudo scalar conclude with transforming law for reflection even number of axial coordinate and appear just in form possibly interaction law with other fields [7].

Lagrangian Effective scalar field equation written in form:

\[
\mathcal{L} = \frac{1}{2} \frac{\partial \varphi}{\partial x^k} \frac{\partial \varphi}{\partial x^k} - \frac{m^2}{2} \varphi^2(x). 
\]

(3)

Correspondence Klien-Gorden equation:

\[
\frac{\partial^2 \varphi}{\partial x^k \partial x_k} + m^2 \varphi(x) = 0. 
\]

(4)

\( \varphi(x) \)- field function

Complex scalar field explain by complex function.

\[
\varphi^*(x) = \varphi_1(x) + i \varphi_2(x). 
\]

(5)

4. THE EFFECT OF COSMIC VACUUM ON SOLUTION OF SCALAR FIELD EQUATION:

Scalar field equation in general form [6]:

\[
\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\nu} \left( \sqrt{-g} g^{\mu\nu} \varphi \right) + m^2 \varphi = 0. 
\]

(6)

In the first we looking for solution this equation in the space-time plane with metric

\[
ds^2 = dt^2 - dx^2 - dy^2 - dz^2. 
\]

(7)

By using this metric, Eq. 6 written in form:

\[
\varphi_{,tt} - \Delta \varphi + m^2 \varphi = 0. 
\]

(8)

Solution of Eq. 8:

\[
\varphi(x,y,z,t) = A_1 e^{i(kx - \omega t)} + A_2 e^{i(kx + \omega t)}, A_1 A_2 \text{ - constant.} 
\]

(9)

We can write Eq. 9 in form:

\[
\varphi(x,y,z,t) = A_1 \cos(kx - \omega t) + A_2 \cos(kx + \omega t). 
\]

(10)
This solution is periodic function. Now, we are looking for solution of Eq. 6 using De Sitter metric.

\[ ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \tag{11} \]

Where

\[ a(t) = e^{\mp Ht} \tag{12} \]

In case the expansion of universe, we use metric in form:

\[ ds^2 = dt^2 - e^{2Ht}(dx^2 + dy^2 + dz^2), -\infty \leq t \leq \infty \tag{13} \]

We’re searching to solve Eq. 6 in form:

\[ \varphi(x, y, z, t) = u(t) e^{ik\vec{x}} = k_1x + k_2y + k_3z. \tag{14} \]

To \( u(t) \) we find:

\[ \ddot{u} + 3H\dot{u} + (k^2e^{-2Ht} + m^2)u = 0. \tag{15} \]

Now we’re looking for accurate solution to Eq. 15, by using new arguments

\[ u(t) = V(\xi), \xi = e^{-2Ht}, \infty \geq \xi \geq 0. \tag{16} \]

We find

\[ V''\xi^2 - \frac{1}{2}\dot{V}\xi + \left(\frac{k^2}{4H^2}\xi + \frac{m^2}{4H^2}\right)V = 0. \tag{17} \]

This is Bessel function and it’s solution in general form:

\[ V = \xi^{\frac{3}{2}} \left\{ C_1 J_\nu \left(\frac{k}{H}\xi^{\frac{1}{2}}\right) + C_2 J_{-\nu} \left(\frac{k}{H}\xi^{\frac{1}{2}}\right) \right\}, \quad C_1, C_2 - const. \tag{18} \]

Where

\[ \nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}. \tag{19} \]

According to sign magnitude \( \frac{9}{4} - \frac{m^2}{H^2} \) we find the following:

\[ v = \begin{cases} 
> 0; & \frac{9}{4} > \frac{m^2}{H^2}, \\
= 0; & \frac{9}{4} = \frac{m^2}{H^2}, \\
\text{imaginary}; & \frac{9}{4} < \frac{m^2}{H^2}.
\end{cases} \tag{20} \]

When \( v = 0 \rightarrow m_0 = \frac{3}{2}H = 2.6 \cdot 10^{-66} \text{g}, m_0 \) – critical mass – mass hypothetic graviton [5].

Now we study the asymptotic representation of solution Eq. 18 considering that expansion of university corresponds \( t \rightarrow \infty \) и \( \xi \rightarrow 0 \).

a. when \( v > 0 \) from Eq. 18 we chose decrease function (first term in Eq. 18):

\[ V = C_1\xi^{\frac{3}{2}} J_\nu \left(\frac{k}{H}\xi^{\frac{1}{2}}\right). \tag{21} \]

For expansion solution model (\( \xi \rightarrow 0 \))

\[ J_\nu \left(\frac{k}{H}\xi^{\frac{1}{2}}\right) \approx \frac{1}{\Gamma(\nu+1)} \left(\frac{k}{2H}\xi^{\frac{1}{2}}\right)^\nu. \tag{22} \]

Therefore we find

\[ V \approx C\xi^{\frac{3}{2}} \nu, \text{where } C = C_1 \frac{1}{\Gamma(\nu+1)} \left(\frac{k}{2H}\right)^\nu = const. \tag{23} \]
Comeback to Eq. 16 we find:

$$u(t) \approx C e^{-\left(\frac{3}{2}+\nu\right)Ht}.$$  \hfill (24)

This function non-periodic, it is decrease regularly with time.

For $\xi \to \infty$ we find the following expression:

$$J_\nu \left(\frac{k}{H} \xi^{1/2}\right) \approx \sqrt{\frac{2H}{\pi k}}\xi^{-\frac{1}{4}} \cos \left(\frac{k}{H} \xi^{1/2} - \frac{\nu \pi}{2} - \frac{\pi}{4}\right).$$  \hfill (25)

Considering Eq. 21 we find:

$$\mathcal{V} \approx D \xi^{\frac{3}{2}} \cos \left(\frac{k}{H} \xi^{1/2} - \frac{\nu \pi}{2} - \frac{\pi}{4}\right),$$

where $D = C_1 \sqrt{\frac{2H}{\pi k}} = \text{const}.$ \hfill (26)

Therefore

$$u(t) \approx D e^{-Ht} \cos \left(\frac{k}{H} e^{-Ht} - \frac{\nu \pi}{2} - \frac{\pi}{4}\right).$$  \hfill (27)

This function increase and fluctuate with time.

b. when $\nu = 0$

$$\mathcal{V} = C_1 \xi^{\frac{3}{2}} J_0 \left(\frac{k}{H} \xi^{1/2}\right).$$  \hfill (28)

To $\xi \to 0$

$$J_0 \left(\frac{k}{H} \xi^{1/2}\right) \approx 1.$$  \hfill (29)

To $u(t)$ we find:

$$u(t) \approx C_1 e^{-\frac{3}{2}Ht}.$$  \hfill (30)

To $\xi \to \infty$

$$J_0 \left(\frac{k}{H} \xi^{1/2}\right) \approx \sqrt{\frac{2H}{\pi k}} \xi^{-\frac{1}{4}} \cos \left(\frac{k}{H} \xi^{1/2} - \frac{\pi}{4}\right).$$  \hfill (31)

$$\mathcal{V} \approx D \xi^{\frac{3}{2}} \cos \left(\frac{k}{H} \xi^{1/2} - \frac{\pi}{4}\right),$$

where $D = C_1 \sqrt{\frac{2H}{\pi k}} = \text{const}.$ \hfill (32)

Therefore

$$u(t) \approx D e^{-Ht} \cos \left(\frac{k}{H} e^{-Ht} - \frac{\pi}{4}\right).$$  \hfill (33)

c. when $\nu = iq$ (imaginary)

$$\mathcal{V} \approx C_1 \xi^{\frac{3}{2}} J_{iq} \left(\frac{k}{H} \xi^{1/2}\right).$$  \hfill (34)

To $\xi \to 0$

$$\mathcal{V} \approx C \xi^{\frac{3}{2}} \frac{iq}{\xi^{1/2}}.$$  \hfill (35)

$$u(t) \approx C e^{-\frac{3}{2}Ht}.$$  \hfill (36)

We can write it in another form “real part”:

$$u(t) \approx C e^{-\frac{3}{2}Ht} \cos(qHt).$$  \hfill (37)
This function decrease and fluctuate with time.

To $\xi \to \infty$

$$V \approx D\frac{1}{\xi^2} \cos\left(\frac{k}{H} \frac{1}{\xi^2} - \frac{iq\pi}{2} - \frac{\pi}{4}\right). \tag{38}$$

$$u(t) \approx D e^{-Ht} \cos\left(\frac{k}{H} e^{-Ht} - \frac{iq\pi}{2} - \frac{\pi}{4}\right). \tag{39}$$

Eq. 39 increase and fluctuate with time.

5. CONCLUSIONS:

In the space-time plane, the scalar field equation has periodic solution. In gravitational field for expansion of university for $t \to +\infty$ the solution of scalar field equation will decrease regularly or decreasing fluctuate according to sign of the difference $\frac{g}{4} - \frac{m^2}{H^2}$, and when $t \to -\infty$ we find increasing fluctuate solution.

References